Prelim Examination 2005 / 2006
(Assessing Units 1 & 2)

MATHEMATICS

Advanced Higher Grade

Time allowed - 2 hours

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used in this paper.**
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} 7x + 3y + 4z &= 29 \\ 10x + 2y + 3z &= 26 \\ 9x + 4y + 5z &= 37 \end{aligned}$$

(5)

2. (*a*) Differentiate $(\tan^{-1} 5x)^6$ (3)

(*b*) Given that $f(x) = \ln\left(\frac{x-1}{x+1}\right)$, obtain and simplify $f'(x)$ (4)

3. The coefficients of x and x^2 in the expansion of $(a + bx)^7$ are 218750 and 262500 respectively. Find the value of the constants a and b. (5)

4. Given that $4 - 3i$ is a root of the equation $z^4 - 12z^3 + 62z^2 - 140z + 125 = 0$, find all the solutions. (6)

5. (*a*) Express $\frac{2x^2 + x + 10}{x^3 + 2x^2 + 4x + 8}$ in partial fractions. (3)

(*b*) Hence evaluate $\int_0^2 \frac{2x^2 + x + 10}{x^3 + 2x^2 + 4x + 8}\, dx$ (5)

6. If the parametric equations of a curve are $x = 4\cos\theta - 5\sin\theta$ and $y = 5\cos\theta + 4\sin\theta$,

evaluate $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$. (5)

7. Use the substitution $4x = 3\sin\theta$ to obtain $\int_0^{\frac{3}{4}} \sqrt{9 - 16x^2}\, dx$ (7)

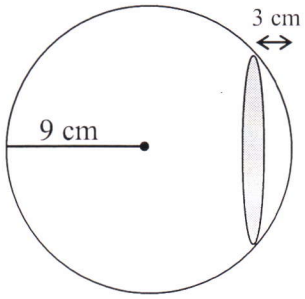
8. The function f is defined by $f(x) \frac{6}{(x+1)(x-3)}$ where $x \in R,\ x \neq -1, 3$.

(a) Find the equations of the asymptotes of the graph of $f(x)$. (3)

(b) Prove that the graph of $f(x)$ has a turning point and determine its nature and coordinates. (5)

(c) Sketch the graph of $f(x)$ indicating all the important features. (2)

9. A sphere has a radius of 9 cm.
A spherical cap of depth 3 cm, is removed from the sphere, as shown.



Using integration methods, find the volume of the cap and hence, or otherwise, find the ratio of the volume of the cap to the volume of the whole sphere. (10)

[**volume of sphere**, $V = \frac{4}{3}\pi r^3$]

10. (a) $x+1,\ 3x+1,\ 6x-2$ are the first three terms of an arithmetic sequence.
For what value of n does S_n, the sum of the first n terms, first exceed 100? (4)

(b) The sum of the first three terms of a positive geometric sequence is 315 and the sum of the 5^{th}, 6^{th} and 7^{th} terms is 80640. Identify the sequence. (6)

[END OF QUESTION PAPER]